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Information in regard to errors of classification or omission will be gratefully received.

BOOK REVIEW.

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Projective Geometry. By L. Wayland Dowling. McGraw-Hill Book Company, New York, 1917. 215 pages. $2.00.

If long and expectant waiting for the guest's arrival insures a hearty welcome, this little book first sees the light under most auspicious circumstances. For many years those who have had the good fortune to teach projective geometry have been wishing for a text in English that should lay sufficient emphasis on its non-metrical character and at the same time should be adapted to the powers of the average college junior or even the exceptional younger student. Cremona, excellent book though it is, does not satisfy the first condition; Holgate's translation of Reye has long been out of print; and Veblen and Young's masterful treatise has seemed to many too heavy for the purpose. Here we have a book, small and compact, of pleasing external appearance, well printed in good type, with clear and attractive page broken by figures of reasonable size. Its point of view is non-metrical and yet it does not neglect the metrical applications. It merits careful consideration by all who are interested in this beautiful field of mathematical thought and sympathetic trial in many of the institutions where the subject is taught.

Perhaps the first question confronting the author of an elementary text on projective geometry is whether the conic shall be studied first as generated by projective ranges and pencils or as made up of the self-conjugate elements of a polar field. Shall he ally himself with the school of which we may take Reye as an example or with that typified by Enriques? The logical advantages of the latter have been clearly pointed out,[1] its wisdom from a pedagogical standpoint is not so clear. It takes much longer to reach what to the student seems worth while.

The teacher finds himself, perhaps, in the position of the guide who must choose between two paths up the mountain. There is on the one hand a long and rugged climb, which leads directly to the top, and on which the wonderful landscape bursts suddenly into view in all its beauty; and on the other is an easier path, which half-way up the mountain affords a fine view, if not the equal of that at the higher level. The most hardy of his party would not stop short of the top in any event. Of those who, exhausted, would fail to finish the longer ascent, some, encouraged by the lesser view, will go on to the greater; others, stopping at the half-way house, will at least have something for their labor. Must it not be agreed that unless the party is made up entirely of hardy climbers the wise guide will choose the second route? But is this a true analogy? We confess doubt.

Our author frankly admits in the preface that he chooses Reye for his pattern, and yet, if we may continue the figure, he does not fail to point the way to the higher level. After viewing the beautiful landscape embracing the conic sections and their properties from the level of generation by projective forms, he concludes the book with chapters on projectively related primitive forms of the second kind and polarities in a plane and in a bundle. The question might be raised whether it would have been possible and desirable in one book to give the teacher his choice of paths. Our author has not attempted this. Much of the book would have to be entirely recast if the teacher wished to make his first approach to the conics through the polar field. In particular the involution on a line could not be derived from the involution on a conic.

With what simple means and how quickly are we led to the conics! Only four chapters, 36 pages, are necessary to introduce the student to the fundamental notions, the primitive forms, the theorem of Desargues, the principle of duality, the theorem of perspective quadrangles, the harmonic set, as a preparation for projective one-dimensional primitive forms, the generation of the conic, the theorems of Pascal and Brianchon, and the polar theory of the conic. These are treated in four chapters covering 57 pages. To be sure, the involution is not yet known, nor those properties that depend on it. Three chapters intervene, Chapter IX on the Diameters, Axes, and Algebraic Equations of Curves of the Second Order, Chapter X on Ruled Surfaces of the Second Order, and Chapter XI on Projectively Related Elementary Forms, before in Chapter XII we have the involution defined as a cyclic projectivity of order two. This chapter deals with the theory of the involution and imaginary elements, and the following Chapter XIII with the foci and focal properties. This closes the discussion of one-dimensional forms, the remaining fifty pages being occupied with two-dimensional forms, the two chapters mentioned above.

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[1] Charlotte Angas Scott, *Annals of Mathematics*, 2d Series, Vol. 2, pp. 64–72.

In general outline these first thirteen chapters handle the subject in about the manner that one familiar with Reye would expect. A few features deserve special mention.

In considering the fundamental theorem the author constructs a harmonic scale, calls attention to the fact that harmonic constructions from three points can never yield all the points of a line, though "theoretically we may arrive at a point-row whose points are everywhere dense," and gives a formulation of the Dedekind postulate as here applied. He confesses that the treatment is meager, but it furnishes a starting point for the teacher who considers it advisable to lay stress on the logical foundation of the fundamental theorem.

Defining the involution on a conic as a cyclic projectivity of order two furnishes the author with an occasion for a short discussion of cyclic projectivities in general. The definition of imaginary elements and the solution of certain problems involving their use follow closely the latest edition of Reye's first volume.

Though nearly a quarter of the book is devoted to metrical matters the double ratio receives scant attention.

A more detailed outline of the last two chapters may be in place, since in the selection of material here there is a wider range of possibilities. The definition and determination of perspective and projective transformations of two-dimensional forms are followed by the plane perspectivity, affinity, similitude, and congruence, with a short discussion of the double elements of a collineation. In the last chapter we have the construction and classification of polarities in the plane and bundle; orthogonal and absolute polarity and antipolarity; two polarities in the same plane or bundle with resulting collineation and with application to the cyclic planes and focal axes of cones; quadratic transformations, inversion, circular transformations.

In handling these two chapters the teacher is afforded ample opportunity to leave his impress upon his course. He will wish to amplify the treatment at many points and readjust the emphasis, in order to bring out the advantages of the polar field approach to the conic. One must delay long enough on the summit for the mists to roll away. He will find it necessary even to correct misleading statements such as,[1] "If two planes are collinearly related and have a self-corresponding line, they are in perspective position, or else they are superposed and have in common a sheaf of rays." Evidently what the author means is a line of self-corresponding points rather than a self-corresponding line. Again, in the demonstration of the theorem,[2] "In affinately related planes, the ratio between the areas of corresponding figures is constant," it is implied that corresponding segments bear to each other a ratio that is constant throughout the plane.

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[1] Page 165.

[2] Page 170.

Scattered through the book there are some fifty sets of exercises averaging six exercises to the set. For the formulation of these the teacher will be grateful. We have noticed two of them that are faulty in statement:[1] "Show that in the configuration of Desargues *any* line may be taken as an axis of perspectivity. The two triangles and the center of perspectivity will then be uniquely determined," is not clear;[2] "If a hexagon whose vertices are not coplanar nor its three diagonals concurrent is projected from any point on a line which meets all three of the diagonals, show that the lines projecting the vertices are rays of a cone of the second order," draws the conclusion of Pascal from the hypothesis of Brianchon.

The historical notes are few and do not attempt to sketch the development of the subject.[3] "Chasles, *Géométrie Supérieure,* 1880," may be thought misleading, since the date is not that of the first edition.

While the author, as he himself says, has patterned after Reye, he has by no means given us a mere translation. The language and style are the author's, not Reye's. Some things we think might be better said in the interest of clearness and accuracy. For example,[4] "Like primitive forms are each composed of the same kind of elements," appears in its connection to be a definition. The breaks in the argument for the purpose of dualizing are sometimes disturbingly frequent. For some things we should use other names; throw of points or lines where the author uses range and pencil, two-dimensional forms for forms of the second kind, quadratic transformation for quadric transformation. We see no reason for banishing the diagonal point. In passing may we express the hope that some steps will soon be taken to standardize the nomenclature of this subject. It is certainly not an advantage pedagogically to have several different names for the same thing.

It is a pleasure to the teacher to be able to point out to his class the excellences of good figures. This pleasure is in measure denied to the user of the book we are discussing; for while the figures are in general clear and easily read, the teacher, if he calls attention to them at all, will be forced to remark on the carelessness exhibited in their construction. The inconsistency in the use of small circles surrounding designated points and in the use of dotted lines is astonishing. In each of two figures three lines that should pass through a point form a triangle of considerable size. Very unfortunate is Fig. 120, one of the most pretentious in the book. The crude approximations to ellipses here shown are an offence to even the slightly trained eye. What a pity that a book must carry a blemish of this sort, which might so easily have been avoided!

Let us conclude with the hope that this book will find wide acceptance and accomplish much in bringing the subject to the attention of a larger body of students.

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[1] Ex. 5, page 23.

[2] Ex. 9, page 80.

[3] Ex. 6, page 53.

[4] Page 17.